Nonlinear Tracking Control of 3-D Overhead Cranes Against the Initial Swing Angle and the Variation of Payload Weight

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Abstract—In this brief, we propose a nonlinear tracking control method of 3-D overhead crane systems which works well even in the presence of the initial swing angle and the variation of payload weight. Besides the practical importance of the overhead cranes, this study is also theoretically interesting because four variables (trolley and girder positions, two swing angles) should be controlled using two control inputs (trolley and girder forces). To control such an underactuated system as cranes, a simple proportional-derivative (PD) controller has been normally used. Unlike the conventional regulation control, the newly proposed nonlinear tracking control law further improves the performance and robustness, which is based on the feedback linearizing control by using the swing angular rate as well as the swing angle. The proposed nonlinear tracking control law eliminates the nonlinear characteristics of the system and achieves the satisfactory position control and swing suppression, even when the initial swing angle and the variation of payload weight exist. We present the stability analysis and simulation results to demonstrate the practical application of our scheme.

Index Terms—3-D overhead crane, initial swing angle, nonlinear tracking control, underactuated system, variation of payload weight.

I. INTRODUCTION

CRANE systems have been widely used in industrial fields. Position control and swing suppression of payloads are needed for crane control. The behavior of the payload shows similar characteristics of an inverted pendulum as it uses flexible cables for suspending the payload, and the dynamics of a crane system are similar to those of an inverted pendulum. Like the inverted pendulum, the crane system is also an underactuated system in that the number of variables to be controlled (the payload position and the swing angle) is by nature less than that of control inputs (the forces driving the trolley and the girder). Since the crane system shows a stable behavior around its desired equilibrium point as opposed to an inverted pendulum, there have been numerous studies done on the simple proportional-derivative (PD) control law. In practical applications, however, it is necessary not only to have a fast position regulation toward the desired position in an economic sense, but also to suppress the swing motion of the payload since a severe payload swing can cause a payload damage or an accident, and also hinder an accurate placement of the payload. Accordingly, much research has been done on the overhead crane systems.

Studies have been done on controllers for the two degree-of-freedom (DOF) crane using nonlinear feedback [1]–[5], optimal dynamic-inversion-based control [6], and fuzzy logic [7]. In addition, controllers for the 3-DOF crane using nonlinear feedback [8] and root locus and loop shaping [9] have been studied. In these results, however, the system nonlinearities are not sufficiently considered in the controller design and the stability analysis. Using the nonlinear stability analysis, PD [10], [11] and nonlinear PID controllers with feedforward control [12] have been proposed for the crane systems. Also, energy-based controllers have been proposed for 2-DOF and 3-DOF crane systems in [13] and [14], [15], respectively, where the nonlinear terms were included to improve the transient performance and to damp the payload swing oscillation quickly. In practical situations, where the industrial crane system is applied, the varying payloads with different weight are quite common. If the effects of changes in payload weight are not considered, it can result in the misleading results and even unstable response [16]. In addition, when the crane encounters winds, there may be a severe turbulence and this can induce the swing motion at the onset of the crane control. Accordingly, the controller should be able to perform well even with these uncertain conditions, as studied in [1].

Considering the aforementioned points, we propose the nonlinear tracking control method using the swing angle and its derivative feedback as in [8] and [12], in order to reduce the swing motion of the payload significantly and also make the crane system robust against the variation of the payload weight. In this brief, we adopt the 3-DOF overhead crane dynamics in [14] and propose a nonlinear controller for positioning and swing suppression of the payload, since three-dimensional overhead crane systems are normally used in many factories and warehouses. Unlike the regulation control in [14] and [15], the nonlinear tracking control law is proposed. In this study, the resulting stability analysis was performed under much less strict assumptions. Also, considering more practical situation, simulation results were performed by including the estimators of trolley/girder velocities and swing angular rates, and were provided to compare the performance of the proposed controller with a PD controller and energy-based controllers in [14], [15].

The rest of this brief is organized as follows. Section II describes the system dynamics of the 3-DOF crane system. In Section III, a nonlinear control law is designed and also the stability of the overall crane system is analyzed. Section IV shows the simulation results of the proposed method. Finally, conclusions are given in Section V.

II. SYSTEM DYNAMICS

The overhead crane dynamics in [8], [14], [15], which are briefly described in this section, were adopted. Fig. 1 shows
In the above expressions, \( m_p \), \( m_g \), and \( m_t \) are the payload mass, girder mass, and trolley mass, respectively; \( I \) is the mass moment of inertia of the payload; \( L \) is the length of the suspending rope; \( q \) is the gravitational acceleration and \( u = [F_x, F_y, 0, 0]^T \). In addition, \( M(q) \) and \( V_m(q, \dot{q}) \) satisfy the skew-symmetric relationship \( \dot{\Omega}^{(1/2)}(M(q) - V_m(q, \dot{q})) = 0 \) and the inequalities \( k_1 ||\dot{\xi}||^2 \leq \dot{\xi}^T M(q) \dot{\xi} \leq k_2 ||\dot{\xi}||^2 \) for \( \dot{\xi} \in R^4 \) and \( M(q) \), which is the time derivative of \( M(q) \).

For the design of the decoupling control law, \( x \) - and \( y \) - dynamics in (1) can be modified as

\[
\ddot{q} = M^{-1}(q)(u - V_m(q, \dot{q}) \dot{q} - G(q))
\]

(2)

which can be rewritten as

\[
\dot{\bar{q}} = \frac{1}{\det(M)}(PF + W)
\]

(3)

where

\[
r = [x \ y \ \theta \ \phi]^T
\]

\[
P = \begin{pmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{pmatrix} > 0
\]

\[
F = [F_x \ F_y]^T
\]

\[
W = [w_1 \ w_2]^T
\]

- \( \det(M) \) stands for the determinant of \( M(q) \)
- \( p_{11} = m_p^2 L^2 (\sin^2 \phi + 2 \sin \theta \cos^2 \phi) + m_p L^2 \)
- \( p_{12} = m_p L^4 \sin^2 \theta \phi \sin^4 \theta \)
- \( p_{22} = m_p L^4 \sin^2 \theta \phi \cos^4 \theta + m_p^2 L^2 I \)

\[
m_{13} = m_p L \sin \theta \cos \phi
\]

\[
m_{24} = -m_p L \sin \theta \sin \phi
\]

\[
m_{31} = m_p L \cos \theta \sin \phi
\]

\[
m_{32} = m_p L \cos \theta \cos \phi
\]

Fig. 1. 3-D overhead crane system.
\[ w_2 = m_p L \sin \theta \cos \phi \]
\[ + [(m_p + m_g + m_t) I + (m_g + m_e) m_p L^2 \sin^2 \theta] \cdot \left[ \frac{\ddot{x}}{m_p L^2 \sin^2 \theta + I} \right] \]
\[ + \frac{\ddot{x}}{m_p L^2 \sin^2 \theta + I} + 2m_p L \ddot{\theta} \cos \theta \sin \phi \]
\[ + [(m_p + m_g + m_t) (m_p L^2 + I) - m_p L^2 \cos^2 \theta] \cdot \left[ \frac{\ddot{y}}{m_p L \sin \theta} \right] \cdot \left[ \frac{\ddot{y}}{m_p L \sin \theta} \right] \]
\[ + (m_p + m_g + m_e) m_p L \cos \theta \cos \phi \cdot \left[ \frac{\ddot{y}}{m_p L \sin \theta} \right]. \]

Also, \( \theta \)- and \( \phi \)- dynamics in (1) can be obtained as

\[
\begin{pmatrix} \dot{\theta} \\ \dot{\phi} \end{pmatrix} = \begin{pmatrix} m_{33} & 0 \\ 0 & m_{44} \end{pmatrix}^{-1} \left\{ \left[ \begin{pmatrix} m_{31} & m_{32} \\ m_{41} & m_{42} \end{pmatrix} \right] \cdot \left[ \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} \right] \right\}.
\] (4)

\[ \gamma(t) > 0 \]

\[ \kappa(x) \]

\[ \gamma(t) = \begin{cases} \frac{1}{\alpha + \beta \varepsilon}, & \text{if } 0 \leq t \leq T_f \\ \frac{1}{1 + \varepsilon}, & \text{if } t > T_f \end{cases} \] (12)

\[ \text{Remark 1: Compared with (7), (8) contains } (\sin \phi \cos \phi)^T \cdot f \]

\[ \text{and uses } [P/\det(M)]^{-1} \text{ instead of } [\Omega]^{-1}. \] By introducing these terms, we could improve the transient performance and robustness, as will be shown in Section IV. The stability and performance of the proposed control law are stated in the following theorem.

**Theorem 1:** Consider the plant (3)–(4) with \( \theta(0) < \pi/2 \), and the control law given by (8)–(12). Then, the trolley and girder position errors \( x_e \) and \( y_e \), and the swing angle \( \theta \) converge to zero asymptotically. Also, \( \dot{q} = [\dot{x} \ \dot{y} \ \dot{\theta} \ \dot{\phi}]^T \) remains bounded for all \( t \geq 0 \).

**Proof:** By substituting (8) into (3) and rearranging the terms, the position error dynamics become

\[ \dot{e} + 2k_e \dot{e} + k_e^2 e = \left( \frac{\sin \phi}{\cos \phi} \right) \cdot f \] (13)

\[ \text{where } e = r - r_d, \text{ in which the target trolley and girder positions, } x_d \text{ and } y_d, \text{ are constants. The } \theta \text{- dynamics in (4) become}
\]

\[ (m_p L^2 + I) \ddot{\theta} + (\cos \theta \sin \phi \ddot{x}_e + \cos \phi \cos \theta \ddot{y}_e) m_p L \]
\[ - m_p L^2 \sin \theta \cos \phi \dot{\phi}^2 + m_p L \sin \theta = 0 \] (14)

\[ \text{or}
\]

\[ (m_p L^2 + I) \ddot{\theta} + \left\{ \sin \phi (\ddot{x}_d - 2k_e \dot{x}_e - k_e^2 x_e) + \cos \phi (\ddot{y}_d - 2k_e \dot{y}_e - k_e^2 y_e) + f \right\} \cdot \cos \theta m_p L \]
\[ - m_p L^2 \sin \theta \cos \phi \dot{\phi}^2 + m_p L \sin \theta = 0. \] (15)

\[ \text{It is noted that only (13) and (15) are used in the control law design, because the } \phi \text{-dynamics become insignificant if } \theta \text{ converges to zero (see Remark 6).} \]
First, a positive definite function for the $x$- and $y$-dynamics is considered as follows:

$$V_1 = \frac{(\dot{e} + k_c e)^T (\dot{e} + k_c e)}{2}. \quad (16)$$

Then, the time derivative of (16) using (13) becomes

$$\dot{V}_1 = (\dot{e} + k_c e)^T (\dot{e} + k_c e) = (\dot{e} + k_c e)^T \left\{ -k_c \dot{e} - k_c e \right\} + \left( \sin \phi \right) \cdot f \nonumber = -k_c (\dot{e} + k_c e)^T (\dot{e} + k_c e) + \left( \sin \phi \right) (\dot{x}_e + k_e \dot{x}_e) + \cos \phi (\dot{y}_e + k_e \dot{y}_e) \leq -k_c (\dot{e} + k_c e)^T (\dot{e} + k_c e) + \left( \sin \phi \right) (\dot{x}_e + k_e \dot{x}_e) + \left( \cos \phi \right) (\dot{y}_e + k_e \dot{y}_e) \leq \gamma(t), \quad (17)$$

This shows the input-to-state (ISS) stability [17] of the e-dynamics with respect to the input $f$, which, in turn, implies the convergence of $\dot{e} + k_c e$ to zero (i.e., of $e$ to zero) when $f$ decreases to zero.

Also, the sway dynamics, (15), are expressed as

$$\dot{\theta}_1 = \theta_2$$

$$\dot{\theta}_2 = -\left( \sin \phi \right) (\dot{x}_e + k_e \dot{x}_e) + \cos \phi (\dot{y}_e + k_e \dot{y}_e) - f + L \sin \theta_1 (\dot{\phi})^2 \quad (18)$$

where $\theta_1 = \theta$, $\theta_2 = \dot{\theta}$, and $\dot{\varphi} = mpL/(mpL^2 + I)$.

Another Lyapunov function candidate for system (18), i.e., the $\theta$-dynamics is considered as follows:

$$V_2 = \frac{1}{2} \theta_2^2 + \frac{1}{2} (k_\theta \theta_1 + \theta_2)^2 + \left( 1 - \cos \theta_1 \right) \quad (19)$$

Using (18), the time derivative of (19) becomes

$$\dot{V}_2 = \theta_2 \dot{\theta}_2 + (k_\theta \theta_1 + \theta_2) \ddot{\theta}_2 + 2 \sqrt{\alpha} \sin \theta_1 \dot{\theta}_1 = (k_\theta \theta_1 + \theta_2) \ddot{\theta}_2 + 2 \sqrt{\alpha} \sin \theta_1 \dot{\theta}_1$$

$$= - (k_\theta \theta_1 + \theta_2) \ddot{\theta}_2 + 2 \sqrt{\alpha} \sin \theta_1 \dot{\theta}_1$$

$$= - (k_\theta \theta_1 + \theta_2) \ddot{\theta}_2 + 2 \sqrt{\alpha} \sin \theta_1 \dot{\theta}_1$$

$$= - (k_\theta \theta_1 + \theta_2) \ddot{\theta}_2 + 2 \sqrt{\alpha} \sin \theta_1 \dot{\theta}_1$$

$$= - \left( \sin \phi \right) (\dot{x}_e + k_e \dot{x}_e) + \cos \phi (\dot{y}_e + k_e \dot{y}_e)$$

$$= - \left( \sin \phi \right) (\dot{x}_e + k_e \dot{x}_e) + \cos \phi (\dot{y}_e + k_e \dot{y}_e)$$

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If we substitute $f$ in (9) into the above equation and use the property $\exists \tilde{K}(\tilde{x}) = \tilde{K}(\tilde{x}) \geq 0$, then we can obtain the following inequality,

$$\dot{V}_2 \leq - k_\theta \sin \theta_1 (\theta_1) + \frac{(k_\theta \theta_1 + 2 \theta_2)^2}{2} \nonumber$$

Then, we can see from (11) that when $|k_\theta \phi + \dot{\theta}| \geq \gamma(t)$, (20) leads to $\dot{V}_2 \leq - k_\theta \sin \theta_1 (\theta_1) + \frac{(k_\theta \theta_1 + 2 \theta_2)^2}{2} \nonumber$ Thus, $\theta$ and $\dot{\theta}$ can be shown to have the asymptotic convergence to the set $(\{\theta, \dot{\theta} \} : |k_\theta \phi + \dot{\theta}| \leq \gamma(t))$, which becomes sufficiently small as time goes on. That is, from (12) $\gamma(t)$ decreases to zero as $t \to \infty$ (i.e., the ultimate bounds of $\theta$ and $\dot{\theta}$ decrease to zero), and thus, $\theta$ and $\dot{\theta}$ in (18) become asymptotically stable.

Using the boundedness of $\dot{x}_e$ and $\dot{y}_e$, $\dot{x}_e$ and $\dot{y}_e$ can be shown to be bounded from (13) and the foregoing arguments. As $(\sin \theta \cos \phi \dot{\phi} - \sin \phi \sin \phi \dot{\phi} \dot{\phi})$ are bounded and converges to zero due to the convergence of $\theta$ to zero, and the $\phi$-dynamics in (4) can be given by

$$(mpL^2 \sin \theta + I) \dot{\phi} + (\sin \theta \cos \phi \dot{\phi} - \sin \phi \sin \phi \dot{\phi}) \dot{\phi} = 0 \quad (21)$$

It can be guaranteed that

$$\frac{d}{dt} (mpL^2 \sin \theta + I) \dot{\phi} \geq 0 \quad (22)$$

converges to zero, i.e., $\dot{\phi}$ becomes bounded. Thus, the control input remains bounded as it can be seen in (9).

Thus, if we consider the overall system as an interconnected system consisting of $e$-dynamics and $\theta$-dynamics, then $e$ and $\theta$ are guaranteed to be stable. Furthermore, since $f$ decreases to zero as $\theta_1$ and $\theta_2$ converges to zero, $e$ becomes asymptotically stable by using the ISS-stability of the $e$-dynamics from (17) [17].

**Remark 2:** Theorem 1 shows that $\theta$ and $\dot{\theta}$ becomes sufficiently small as time goes on, in which case the terms $f$ and $W$ in the control input (8) reduce to zero, and thus, the control input (8) becomes a PD control law in (6). Thus, we can say that the proposed nonlinear control law is a generalization of PD control law, which considers the nonlinear characteristics of the crane system and also the swing angle motion.

**Remark 3:** The robustness of the proposed method against the initial swing angle can be discussed as follows. It should be noted that Theorem 1 does not require $\gamma(t)$ or $\dot{\gamma}(t)$ as in [12] or $\dot{\gamma}(t) = 0$ as in [14], [15] for the asymptotic convergence of $x_e, y_e, \theta$ to zero. In the case of $\theta(t) \neq 0$, although $F = 0$ (or $u = 0$), $\dot{x}(t) = 0$ and $\dot{y}(t) = 0$ may not be zero. This means that the approaches in [12] or [14], [15] do not have the guaranteed performance against the initial swing angle, unlike the proposed method (see Fig. 3 where the robustness against the initial swing angle is shown). In particular, the stability analysis in [15] assumes the desired positions to be constants and relies on the LaSalle’s invariance principle, which is not the case with this brief (see Fig. 5 where the trajectory tracking performance is shown). Thus, the tracking control result with the robustness against the initial swing angle in this brief can be said to be more general than the previous regulation results.

**Remark 4:** The robustness of the proposed method against the variation of the payload weight can be discussed as follows. Since the payload weight changes in practical situation, the uncertainty exists in the payload weight. For example, actual payload mass $m_p$ and nominal payload mass $m_p$ can be related as $m_p = (1 + \delta)m_p$, where $\delta$ is an uncertain constant parameter.
satisfying $\delta > -1$. For the simplicity of the analysis, we assume the followings: i) since $I$ is sufficiently small and can be neglected, $m_{33}$ and $m_{44}$ become almost linearly dependent on $m_p$; ii) the masses of the payload, girder, and trolley in $m_{11}$ and $m_{22}$ are related in such a way that the actual girder and trolley masses are $(1+\delta)m_g$ and $(1+\delta)m_t$ just like $m_{ap} = (1+\delta)m_p$. Now, we can see from $M(q)$, $V_m(q, \dot{q})$, and $G(q)$ that the elements of these matrices become linearly dependent on $1+\delta$, and
thus, by considering $1 + \delta = 1/(1 + \Delta)$, the system dynamics with input uncertainty can be given by

$$\dot{\theta} = \frac{PF(1 + \Delta) + W}{\det(M)}$$  \hspace{1cm} (23)

where $\Delta$ is an uncertain constant parameter satisfying $\Delta > -1$. Note that, in the case of $\Delta = 0$, (23) reduces to the nominal system dynamics in (3). The stability analysis as in the proof of Theorem 1 can show that $\theta$- and $\phi$- dynamics are almost the same as (4) even with the variation of the payload weight.
This explains the robustness against the variation of the payload weight (see Figs. 4 and 5).

**Remark 5:** The condition for $|\theta(0)| < \pi/2$ in Theorem 1 was also assumed in [12]. Considering the practical application, this makes sense and does not lose the generality of the proposed controller.

**Remark 6:** The convergence of $x_c$, $y_k$, and $\theta$ to zero is not affected by the $\dot{\theta}$-dynamics. In [13] as well, only the swing ($\theta$) dynamics were investigated, excluding the rotational $\dot{\theta}$-dynamics. This approach makes sense, considering the fact that the rotational angle $\dot{\theta}$ has no meaning if $\theta$ becomes zero.

**Remark 7:** We used the swing angle and swing angular rate in the feedback control (8) to reduce the payload swing as in [8] and [12]. This requires the information of the swing angular rate $\dot{\theta}$ and trolley/girder velocities $\dot{x}, \dot{y}$ as well as the trolley/girder positions and the swing angle. It should be emphasized that in [2], [4], [10]–[12], [15] the nonlinear control laws for crane systems used the full state feedback and the nonlinear observer could not be designed, which is different from the results of linear control laws for crane systems [3], [6], [7] where Luenberger observer can be designed based on the linearized model. Instead, unmeasured states (the trolley/girder velocities and swing angular rate) were estimated by numerical backward difference technique, which is followed by the low-pass filtering. In addition, we used the low-pass filtered values of measured position and rotational angle in the feedback control law, since the measured position and rotational angle can be contaminated with high-frequency noise, and in particular, unwanted high frequency payload sway oscillations due to the cable vibration can be measured by the optical encoder [18]. We could see from the simulation results in the next section that the proposed method works well even in this situation.

**Remark 8:** To reduce the chattering phenomenon [17] and also avoid the unlimited control input problem in adaptive variable structure control [19]–[21], a function $\kappa(\cdot)$, which is defined by (11) as in [22], is used in (9) instead of a signum function.

### IV. SIMULATION RESULTS

This section presents simulation results for the proposed nonlinear control law using the 3-DOF overhead crane model in [14], [15]. The system parameters in (1) were selected as

$$m_p = 0.73 \text{ kg} \quad m_t = 1.06 \text{ kg} \quad m_g = 3.0 \text{ kg} \quad I = 0.005 \text{ kg m}^2 \quad L = 0.7 \text{ m}.$$  

As in [14], the design parameters of the PD controller in (6) are $k_d = 102$, $k_p = 45$, and $k_E = 1$, and those of the $E^2$ controller in (7) are $k_d = 125$, $k_p = 50$, and $k_E = 0.001$. Finally, the design parameters of the proposed controller in (8)–(12) were $k_c = 1$, $k_\theta = 0.1$, $\alpha = \beta = \eta = 0.5$, and $T_f = 10$. The control inputs $F_x, F_y$ are constrained to a maximum value of $\pm 60 \text{N}$ [refer to Figs. 2–5(d)]. The unmeasurable state variable $\hat{q}$ is estimated by using the backward difference technique, which along with the measurable state variable $q$, is filtered by the discrete low-pass filter $0.8/(1 - 0.2s^{-1})$ by considering the practical issues discussed in Remark 7.

The crane system is known to move payloads with different weight and also the initial swing angle $\theta(0)$ can be different from zero due to various factors. Accordingly, the actual payload mass $m_{ap}$ can be different from the nominal payload mass $m_p$. Thus, $m_p$ and $m_{ap}$ were used in the controller and the plant, respectively. The controller should show satisfactory performance for the variation of the payload weight $m_{ap}$, and the non-zero initial swing angle $\theta(0)$. The position errors $x_c$, $y_k$, and the swing angle $\theta$ were chosen as performance indices. The performance of both regulation and tracking control is evaluated in Figs. 2–4 and 5, respectively.

First, the regulation performance of the several controllers was evaluated for $m_{ap} = 0.73 \text{ kg}$ ($m_{ap}/m_p = 1$) and $\theta(0) = 0 \text{ deg}$. The desired position was chosen as $[x_d, y_d]^T = [1.75(m), 1.75(m)]^T$. Note that the performance of the controller was similar for other desired positions (e.g., $[x_d, y_d]^T = [-2(m), 1(m)]^T$, which is omitted for brevity. In Figs. 2(a) and (b), the proposed controller shows better transient performance, compared with both PD and $E^2$ controllers. In particular, Fig. 2(c) shows that the oscillation of the swing angle occurring in the PD and $E^2$ controllers are significantly and quickly reduced by the proposed controller.

Secondly, the payload mass was the same as the nominal mass $m_p$ (i.e., $m_{ap} = 0.73 \text{ kg}$ or $m_{ap}/m_p = 1$) and the initial swing angle is changed as $\theta(0) = 10 \text{ deg}$. In this case, the performance of the PD controller showed unstable performance due to the large non-zero initial swing angle, and thus, it is omitted here. The proposed controller showed the smaller rise time than the $E^2$ controller, as shown in Fig. 3(a) and (b). In particular, compared to Fig. 2(c), $E^2$ controller showed the longer and larger oscillating swing angle, but the proposed controller showed satisfactory performance of the position control and swing suppression as in Fig. 3(c).

Third, the payload mass was changed to 10% of the nominal mass $m_p$ (i.e., $m_{ap} = 0.073 \text{ kg}$ or $m_{ap}/m_p = 0.1$) and the initial swing angle was $\theta(0) = 0 \text{ deg}$. Although PD and $E^2$ controllers showed the position regulation performance as anticipated, the proposed controller showed better regulation performance in terms of both position error and swing angle, as can be seen in Fig. 4.

Finally, the tracking control performance of the proposed controller is shown in Fig. 5 with no payload variation and initial swing angle. In addition, the desired position was chosen as $[x_d, y_d]^T = [1.75 + 0.1t(m), 1.75 + 0.1t(m)]^T$, which increases with time of slope 0.1 from $x_d(0) = y_d(0) = 1.75(m)$. The tracking control performance of PD, $E^2$, and the proposed controllers is included in Fig. 5. As analyzed in the proof of the Theorem 1, we can obtain the expected performance in terms of position tracking errors and swing angle.

From the simulation results under various conditions, we can see the advantage of the proposed method, in the sense that: 1) the better transient response in the case of no initial swing angle and the variation of swing angle and 2) the robustness against the initial swing angle and the variation of payload weight can be obtained further. That was similar even in the presence of...
uncertainties in the rope length \( L \), the payload inertia, trolley/girder masses, the details of which are omitted.

V. CONCLUSION

We proposed a nonlinear tracking control law for the 3-DOF crane system. By using the feedback terms of the swing angle and the swing angular rate, we could control the position of the crane system and also effectively suppress the swing motion of the crane system. The performance of the proposed controller was compared with that of the PD controller and the previous energy-based controller, showing the robustness of the proposed control system for the initial swing angle and the change of the payload weight. In particular, we performed the stability analysis of the tracking control under less strict assumptions, compared with the previous regulation results. In order to consider the practical aspect of \( J \), the hoist control of the 3-DOF crane system needs to be pursued further as a future work.

REFERENCES